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Scale-Bridging Numerical Methods

The list below includes possible topics and the necessary literature. Modifications and extensions are generally possible. Access to the literature is provided if required.

1 Super-time-stepping for parabolic problems

This talk is based on [AAG96]. The talk should explain the methodology and present exemplary numerical simulations.

2 One-dimensional adaptive finite elements

This talk is based on [Ver96, Sec. 1.1-1.2]. The focus of this topic is a practical adaptive finite element implementation in one dimension using residual-based error estimators and the Dörfler marking strategy. A possible numerical example is presented in [AOY21, Ex. 1].

3 Error estimators based on solving local problems

This talks follows [Ver96, Sec. 1.3]. The focus is the theoretical understanding of the construction of the error estimators.

4 *hp*-finite element method for high-frequency Helmholtz problems in one dimension

This talk is based on [Ihl98, Sec. 4.7], with Sec. 4.6 being important background on the pollution effect. It should cover the method's introduction (Sec. 4.7.1) and its analysis in (Secs. 4.7.2, 4.7.5, and 4.7.6). Exemplary numerical simulations in the high-frequency regime should also be included.

5 Multigrid solvers for discretizations of diffusion-type problems

This talk follows [BS08, Ch. 6], introducing the multigrid algorithm and its analysis as presented in Secs. 6.5 and 6.6. It should also include an implementation of the multigrid method in a simplified setting.

6 BPX as additive variant of multigrid for diffusion-type problems

This talk follows [BS08, Ch. 7], beginning with the general framework of additive Schwarz methods (Sec. 7.1), and then presenting and analyzing the BPX method within that framework (Sec. 7.3) (using some results from Sec. 7.2). An implementation of the BPX preconditioner in a simplified setting should also be included.

7 Flux-corrected transport for one-dimensional hyperbolic problems

This talk is based on [KH23, Ch. 2], covering the discretization of one-dimensional hyperbolic problems (Sec. 2.1) and the flux-corrected transport method for discretely preserving physical properties of the solution (Sec. 2.2). Numerical illustrations in one dimension should also be included.

8 One-dimensional homogenization

This talk is based on [MP20, Ch. 2], particularly Secs. 2.1 and 2.2. Apart from the theory, this talk should include numerical illustrations of the behavior of finite elements for oscillatory problems and the effect of homogenization.

9 Localized orthogonal decomposition in one dimension

This talk is based on [MP20, Sec. 2.4] (Sec. 2.1 should be read for an introduction). Numerical examples that illustrate the performance of the localized orthogonal decomposition should complement the theory.

Literatur

[AAG96] V. Alexiades, G. Amiez, and P.-A. Gremaud. Super-time-stepping acceleration of explicit schemes for parabolic problems. *Communications in Numerical Methods in Engineering*, 12(1):31–42, 1996.

[AOY21] C. O. Akeremale, O. A. Olaiju, and S. H. Yeak. *h*-adaptive finite element methods for 1D stationary high gradient boundary value problems. *Mathematical Modelling of Engineering Problems*, 8(6), 2021.

[BS08] S. C. Brenner and L. R. Scott. *The Mathematical Theory of Finite Element Methods*. Springer, 2008.

[Ihl98] F. Ihlenburg. *Finite Element Analysis of Acoustic Scattering*. Springer-Verlag, 1998.

[KH23] D. Kuzmin and H. Hajduk. *Property-Preserving Numerical Schemes for Conservation Laws*. World Scientific, 2023.

[MP20] A. Målqvist and D. Peterseim. *Numerical homogenization by localized orthogonal decomposition*. SIAM, 2020.

[Ver96] R. Verfürth. *A review of a posteriori error estimation and adaptive mesh-refinement techniques*. Wiley-Teubner, 1996.